QUARK-ANTIQUARK POTENTIAL FROM INVERSION OF HADRON SPECTRAL DATA

LEEB H.‡, FIEDELDEY H.*, OEHM W.*, SOFIANOS S.A.*, LIPPERHEIDE R.**

‡Institut für Kernphysik, Technische Universität Wien, Austria
*Department of Physics, University of South Africa, Pretoria
**Hahn-Meitner-Institut Berlin, West Germany

Contribution to the International Conference on Hadron Spectroscopy - HADRON89,
23.-27. 9. 1989, Ajaccio, France

supported by Fonds zur Förderung der wissenschaftlichen Forschung in Österreich (Proj.Nr.P6608)

QUARK-ANTIQUARK POTENTIAL FROM INVERSION OF HADRON SPECTRAL DATA+

LEEB H.‡, FIEDELDEY H.*, OEHM W.*, SOFIANOS S.A.*, LIPPERHEIDE R.**

‡Institut für Kernphysik, Technische Universität Wien, Austria
*Department of Physics, University of South Africa, Pretoria
**Hahn-Meitner-Institut Berlin, West Germany

It is a great challenge of modern physics to understand the rich spectroscopy of hadrons in terms of quarks and gluons. Today we believe that QCD is the underlying theory but unfortunately no full solution of its field equations is known. Hence all investigations on the structure of hadrons rely on some model. In particular non-relativistic models have been successfully applied. The concept of a quark-antiquark potential being the basis of such models can only be justified in the limit of heavy quarks. However, surprisingly good results are also obtained for relatively light quark systems.

The experimentally known meson spectra offer the possibility to determine the quark-antiquark potential. In the literature many analyses[1-5)] of the meson spectra are reported using phenomenological or semiphenomenological quark-antiquark potentials. The parameters are adjusted in order to fit the meson masses and sometimes the leptonic widths. Following the qualitative results of lattice gauge calculations[6)] and perturbation theory almost all potentials contain a Coulombic term at small distances and are monotonically increasing at large distances. However, all calculations use rather restricted forms of the potential thereby introducing a bias into its shape.

In this contribution we want to apply the exact inverse spectrum method as introduced by Thacker et al.[7)] for s-state mesons. This method has been used[7)] to analyse charmonium which was the most suitable known $q\bar{q}$-system at that time. It is obvious that the inversion based on only two states (J/Ψ and Ψ(2s)) cannot give a good reproduction of the confinement potential. Here, we analyse the $b\bar{b}$-system which is the best suited one today. The $b\bar{b}$-system has not only a rich known spectrum[8)] but because of the heavy b-quark it is the least relativistic quark system.

Thacker et al.[7] have studied the inverse problem for confining potentials. They consider one-dimensional confining potentials symmetric with respect to the origin. Approximating the spectrum of the potential by N bound states with wave numbers κ_n, n=1,2,...,N and assuming an asymptotic value of the potential E_0 as well as vanishing reflection in the continuum part of the spectrum they can exactly determine a potential which should approximate the original confining one. In the three-dimensional case the radial equation for s-wave scattering can be directly related to the one-dimensional problem with a symmetric potential. The s-wave bound states (vector mesons with mass $M_n c^2$) correspond to the odd-parity states in the one dimensional problem,

$$M_n c^2 = E_0 - \frac{\kappa_n^2}{2\mu c^2}, \qquad n=2,4,...,N \qquad n \text{ even}, \tag{1}$$

where μc^2 is the reduced mass of the $b\bar{b}$-system. The wave numbers κ_n (n odd) of the even-parity states can be determined from the wave function $\Psi(0)$ of the three-dimensional problem which is related to the leptonic width $\Gamma(\Upsilon \rightarrow e^+e^-)$ via the Weisskopf-van Royen formula[9],

$$\Gamma(\Upsilon(ns) \rightarrow e^+e^-) = 16\,\pi \frac{e_q^2\,\alpha^2}{M_n c^2}\, f_{QCD}\, |\Psi_n(0)|^2 \,. \tag{2}$$

Here e_q is the quark charge, α is the fine structure constant, and f_{QCD} takes into account radiative chromodynamical corrections[10]. It has been shown[11] that the knowledge of all masses of s-wave mesons and their leptonic width fix the potential uniquely.

In a realistic system like $b\bar{b}$, however, we know only part of the spectrum. Because of this deficiency we want to include in our inversion procedure the whole available experimental information and to treat also particles of other partial waves. For this purpose we embed the inversion method into a least square fit. We assume the associated one-dimensional problem to have N bound states. The wave numbers κ_2 and κ_4 belonging to the two lowest lying states of odd parity in the one-dimensional problem can be directly obtained by (1) from the masses of $\Upsilon(1s)$ and $\Upsilon(2s)$. The leptonic widths (2) of these particles are used to determine the wave numbers κ_1 and κ_3. The remaining wave numbers $\kappa_5, \kappa_6, ..., \kappa_N$ are free parameters which can be optimized by fitting spectral data from other mesons by minimizing the criterion function

$$\chi^2 = \sum_{i=1}^{M} \frac{[Q_i(\text{exp}) - Q_i(\text{theor})]^2}{\Delta Q_i^2}, \tag{3}$$

where Q_i are the theoretical and experimental masses or leptonic widths of the hadrons, respectively. The weights of the masses in the fitting procedure are given by $\Delta Q_i = \Delta M_i + \Gamma/4$ where ΔM_i are the statistical uncertainties[8]. The total width Γ of the particle state is also included in ΔQ_i in order to account for channel coupling effects which possible would distort a single channel potential. For the weights of the leptonic widths we use their statistical uncertainties[8]. Summarizing our method is a "model-independent" fit procedure which is based on a parametrised form of the potential directly obtained from an exact inverse spectrum theory. This method is more versatile than the exact inversion of Thacker et al.[7], since it allows us to treat all meson data simultaneously. Furthermore the increased number of bound states taken into account improve the simulation of the confining part of the potential as it will be shown in detail in a forthcoming paper[12].

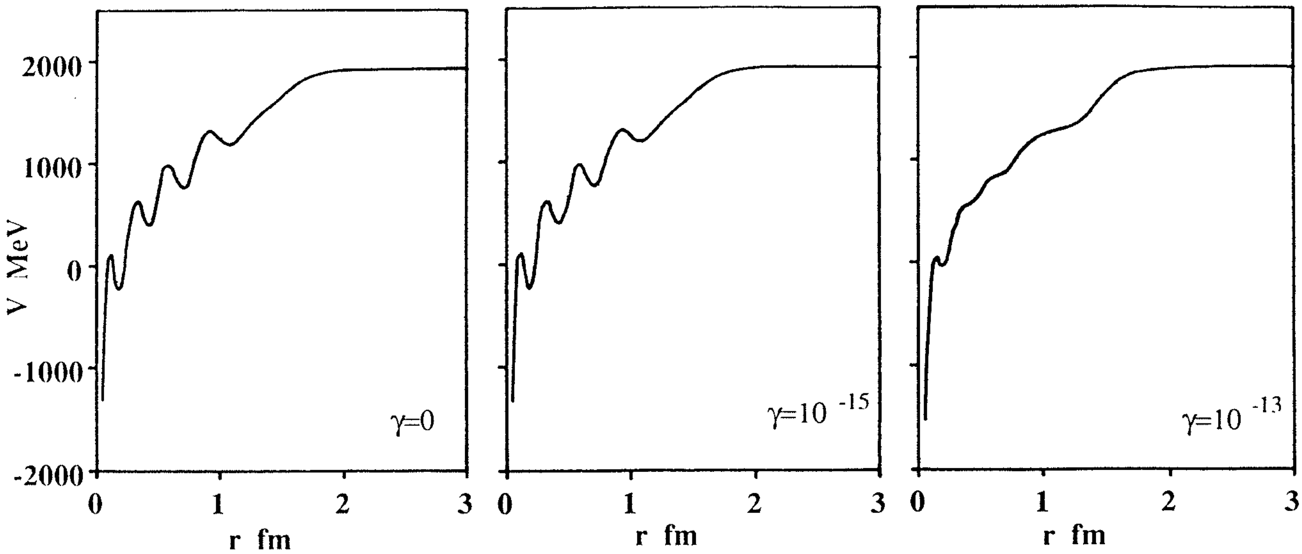

Figure 1
Potential obtained by inverting the bottomonium spectrum using different weights γ for the concavity condition.

As mentioned previously we apply the method to the spectrum of bottomonium[8)] which consists of 8 well established vector particles. With N=12 we get a complete reproduction of all masses and the leptonic widths (assuming[13)] f_{QCD}=1.5) as given in the Table. The resulting potential (Fig.1) has the gross structure of conventional $q\bar{q}$ potentials but exhibits strong oscillations thus violating the concavity condition[14)]

$$\frac{dV}{dr} > 0 \ , \quad \frac{d^2V}{dr^2} \leq 0 \ , \tag{4}$$

which is a general property of static potentials originating from a gauge theory. The origin of the oscillations is twofold: i) the truncation to a finite number N of bound states and ii) the inclusion of states like the ϒ(4s) and ϒ(10860). These states are near the $B\bar{B}$ threshhold and cannot be described by a single channel model.

Table
The energy eigenvalues of the associated one dimensional problem as obtained by fitting the bottomonium masses and leptonic widths for different weights taking into account the concavity condition. All energies are given in MeV.

No.	γ=0	$γ=10^{-15}$	$γ=10^{-13}$		No.	particle	γ=0	$γ=10^{-15}$	$γ=10^{-13}$
1	6469.8	6480.2	6978.4		2	ϒ(1s)	9460.1	9460.1	9460.1
3	9688.5	9688.0	9710.7		4	ϒ(2s)	10022.2	10022.2	10022.2
5	10139.4	10139.6	10149.3		6	ϒ(3s)	10355.4	10355.4	10355.4
7	10446.4	10446.5	10456.3		8	ϒ(4s)	10580.7	10580.8	10580.1
9	10642.2	10642.6	10670.5		10	ϒ(10860)	10868.7	10866.2	10822.8
11	10912.5	10911.5	10919.0		12	ϒ(11020)	11020.0	11019.3	11007.1
E_0	11023.3	11022.6	11020.2		χ^2		3.4654	3.4768	18.6422

In order to overcome the difficulty of the second point we can exclude those states which are strongly affected by coupling effects from the fitting procedure. As a consequence we would retain a too small number of states and would be unable to perform inversions. Therefore we prefer to fit these critical states also, but include the concavity condition as additional information. Following Turchin et al.[15] we formulate an apriori probability

$$P_{apriori} = \exp\{-\chi^2_{apriori}\}, \tag{5}$$

$$\chi^2_{apriori} = \gamma \left| \int_0^\infty dr \, \frac{d^2V}{dr^2} \, \Theta\left(\frac{d^2V}{dr^2}\right) \right|^2 \tag{6}$$

and minimize the quantity

$$\tilde{\chi}^2 = \chi^2 + \chi^2_{apriori} \, . \tag{7}$$

With increasing γ the oscillations become less violent (Fig.1). At the same time the resulting mass for the $\Upsilon(10860)$ is decreasing thus indicating that the too large level distance between the $\Upsilon(4s)$ and the $\Upsilon(10860)$ state (due to coupling effects) is mainly responsible for the oscillations. Although the reproduction of the remaining particle masses and leptonic widths is very good the potential still violates the concavity condition.

The inverse spectrum method yields one special potential out of the family of potentials which can reproduce the masses and leptonic widths of the known particles in the $b\bar{b}$ system. Recently, using a nearly "model-independent" potential model[16] we have shown that the meson masses determine the $q\bar{q}$ potential only in a radial range 0.7fm<r<1.7fm. At present we cannot perform a similar error analysis for the inverse spectrum method because we have too many free parameters compared to the number of states. Therefore strong correlations occur which require an adequate regularisation procedure before statements on the uncertainties can be given.

Following the procedure of ref. 15 we have performed a fit of the masses and leptonic widths in the $b\bar{b}$ system using the modified Cornell potential

$$V(r) = \left(\frac{a}{r} + b\, r^{2/3} + c\right) F(t), \quad F(t) = 1 + \sum_{k=1}^{K} C_k \sin(k\pi t), \qquad t = \left(1 + \frac{r}{r_0}\right)^{-1} . \tag{8}$$

Assuming the weights ΔQ_i as purely statistical, we can determine the error bars for the potential by standard techniques[17]

$$\langle \Delta V(r)^2 \rangle = \sum_{k=1}^{\hat{K}} \sum_{m=1}^{\hat{K}} \frac{\partial V(r)}{\partial a_k}\Big|_{\hat{a}} \frac{\partial V(r)}{\partial a_m}\Big|_{\hat{a}} \varepsilon_{km} \, , \tag{9}$$

where $\hat{K}$ is the total number of parameters varied in the least square fit and $\hat{a}$ denotes the best values of the parameters.The error matrix $\underline{\varepsilon} = (\varepsilon_{km})$ is defined by

$$\underline{\varepsilon} := \underline{\alpha}^{-1} \frac{\chi^2(\hat{a})}{F} \, . \tag{10}$$

Here, $F=M-\hat{K}$ is the number of degrees of freedom and $\underline{\alpha}$ is the curvature matrix

$$\alpha_{km} = \frac{1}{2} \frac{\partial^2 \chi^2(\hat{a})}{\partial a_k \partial a_m}\Big|_{\hat{a}} \, . \tag{11}$$

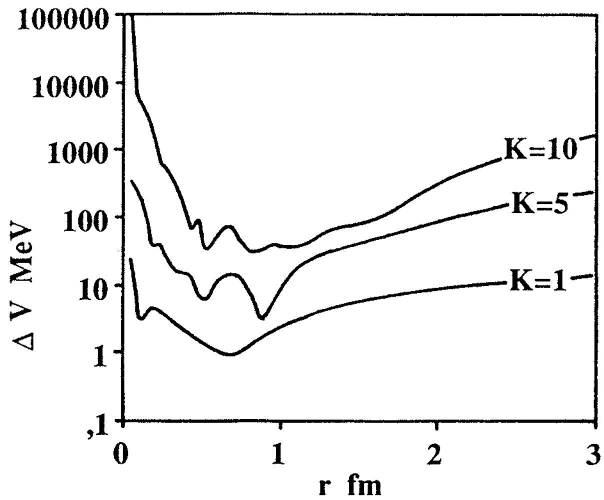

Figure 2
Uncertainties ΔV(r), as defined by Eq. (9), obtained by fitting the bottomonium spectrum to the modified Cornell potential of Eq. (8), using different numbers K of expansion coefficients.

The uncertainties in the potential are shown in Fig. 2 for different numbers K of expansion coefficients. We notice that the potential is only determined in the radial range 0.4fm<r<1.2fm by the experimental data of the bottomonium spectrum.

Assuming a flavour independent quark-antiquark potential we have also performed a fit on the masses of all well established vector mesons[16)] containing only quarks of one flavour. Additional to our previous analysis[16)] we have included in the fit the leptonic widths of the Ψ and ϒ particles forming a set of 34 observables which should be described by the potential model. The uncertainties are shown in Fig. 3 and confirm our previous finding[16)] that the presently known experimental meson spectral data determine the potential in the radial range 0.4fm<r<1.5fm.

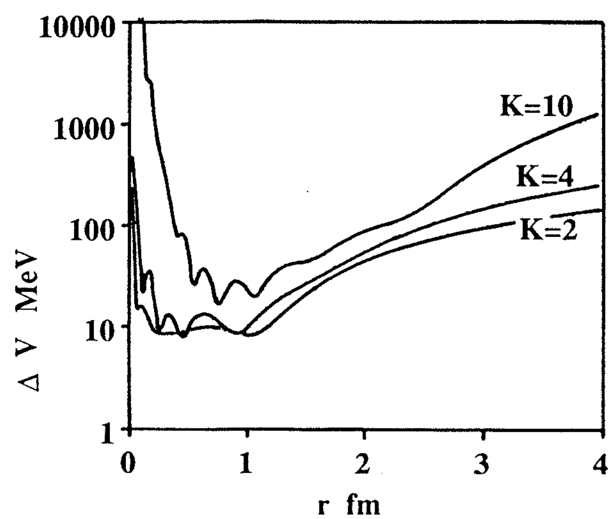

Figure 3
Uncertainties ΔV(r) obtained by fitting all vector meson masses and the leptonic width of Ψ and ϒ to the modified Cornell potential with K expansion coefficients.

In this contribution we have combined the exact inversion method of Thacker et al.[7] with a fitting procedure in order to use the information contained in known states of other partial waves thus improving the representation of the confinement part of the potential. The application to the bottomonium spectrum yields potentials with oscillatory behaviour which violate the concavity condition. Performing a type of statistical regularisation reduces the oscillations due to coupling effects. However, oscillations in the potential, which are caused by the incompleteness of the representation of the spectrum of a confining potential by a finite number of bound states, still remain and limit its application in non-relativistic quark models. The potential obtained by inversion is one member of a family of potentials reproducing the bottomonium data. An error analysis in a potential model which also fits the leptonic widths confirms our previous work based on a fit of masses only that the potential is determined in the radial range $0.4\mathrm{fm}<r<1.5\mathrm{fm}$. This differs substantially from the range $0.1\mathrm{fm}<r<1.0\mathrm{fm}$ usually given[4] by comparing existing potential models with analytically prescribed shapes. Since calculations in perturbation theory are only valid up to about 0.2 fm it is an important consequence of our analysis that the extraction of QCD-related parameters from the meson spectrum requires additional information or assumptions.

References

+) supported by Fonds zur Förderung der wissenschaftlichen Forschung in Österreich (Proj.Nr.P6608)

1) E. Eichten et al., Phys. Rev. **D17**, 3090 (1978); **D21**, 203 (1980).
2) A. Martin, Phys. Lett. **93B**, 338 (1980).
3) J.L. Richardson, Phys. Lett. **82B**, 272 (1979).
4) W. Buchmüller and S.-H.H. Tye, Phys. Rev. **D24**, 132 (1981).
5) D.B. Lichtenberg et al., Z. Phys. C - Part.and Fields **41**, 615 (1989).
6) H.-Q. Ding, Phys. Lett. **200B**, 133 (1988).
7) H.B. Thacker et al., Phys. Rev. **D18**, 274 (1978); **D18**, 287 (1978).
8) Particle Data Group, Review of Particle Properties, Phys. Lett. **204B**, 1 (1988).
9) V. van Royen and W. Weisskopf, Nuovo Cimento **50**, 617 (1967).
10) R. Barbieri, Phys. Lett. **B57**, 455 (1975).
11) H. Grosse and A. Martin, Nucl. Phys. **B148**, 413 (1979).
12) H. Leeb et al., in preparation.
13) A.A. Bykov et al., Usp. Fiz. Nauk **143**,3 (1984) (Sov. Phys. Usp.**27**, 321 (1984)).
14) C. Bachas, Phys. Rev. **D33**, 2723 (1986).
15) V.F. Turchin et al., Usp. Fiz. Nauk. **102**, 345 (1970) (Sov. Phys. Uspekhi **13**, 681 (1971)).
16) H.Leeb et al., Contribution to the Conference Few Body 12, Vancouver, July 1989.
17) P.R. Bevington, Data Reduction and Error Analysis for the Physical Sciences(McGraw-Hill, N. Y., 1975).